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MARANGONI CONVECTION IN A ROTATING FLUID LAYER WITH NON-UNIFORM TEMPERATURE GRADIENT

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Abstract—The onset of Marangoni convection driven by surface tension gradients in a thin horizontal fluid layer is studied by means of linear stability analysis assuming that one of the bounding surfaces is free and adiabatic and the other rigid adiabatic or isothermal. A Galerkin technique is used to obtain the eigenvalues which are then computed numerically. The Coriolis force and inverted parabolic basic temperature profile are suitable for material processing in a microgravity environment for they suppress Marangoni convection considerably. The analytical results compare well with the numerical results of Vidal and Acrivos [*Physics Fluids* **9**, 615–616 (1966)] in the absence of Coriolis force.

NOMENCLATURE

a	dimensionless wave number, $(l^2+m^2)^{1/2}$
A, B	arbitrary constants
$\mathbf{A}$	uniformly bounded body force including accelerations
C	capillary number, $\mu\kappa/\sigma_0 d$
d	thickness of the fluid layer
D	derivative, d/dz
$f(z)$	non-dimensional temperature gradient such that $\int_0^1 f(z)\,dz = 1$
H	dimensionless mean curvature
I	identity tensor
l, m	horizontal wave numbers in the x- and y-directions
M	Marangoni number, $\lvert d\sigma/dT\rvert_{T=T_0}\Delta T d/(\mu\kappa)$
M_c	critical Marangoni number
$\hat{n}$	unit normal vector
p	pressure
Pr	Prandtl number, ν/κ
$\mathbf{q}$	velocity vector
S	free surface
t	time
T	temperature of fluid
T_0	ambient temperature
T_S	surface temperature
ΔT	temperature difference between the boundaries, $T_S - T_0$
$\mathscr{T}^2$	Taylor number, $4\Omega^2 d^2/\nu^2$
u, v, w	components of velocity in x-, y- and z-directions
x, y, z	Cartesian coordinates.

Greek symbols

α	square of wave number, a^2
ε	depth
ζ	z-component of vorticity
η	deflection of the free surface
κ	thermal diffusivity
μ	fluid viscosity
ν	kinematic viscosity, μ/ρ
ρ	fluid density
σ	surface tension
σ_0, σ_1	constants
$\tau =$	$-pI + \nabla\mathbf{q} + \nabla\mathbf{q}^T$ stress tensor
Ω	angular velocity.

Operators

$\langle\ldots\rangle$	vertical average, $\int_0^1 (\ldots)\,dz$
∇_1	gradient in the horizontal plane
∇^2	Laplacian, $\partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$.

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1. INTRODUCTION

A GREAT deal of interest has been evinced recently in the possibility of producing new materials in the reduced gravity (i.e. microgravity) environment usually encountered during sustained space flight [1]. The range of possibilities extends from producing large crystals of uniform properties to manufacturing materials with unique properties. Most of these properties are affected by buoyancy induced convection, called Rayleigh–Bénard convection. Although the microgravity environment prevents this convection, there will be another type of convection arising due to non-uniformities in the surface tension (see refs. [1, 2]), called Marangoni convection. Such convection can still influence the local material composition and can result in solids with non-uniform properties and crystal defects. The mechanism of suppressing such convection plays a vital role in the future material processing in a microgravity environment. It is known that the Coriolis force under proper values of the Taylor number suppresses convection [3]. In addition to this, a suitable non-uniform basic temperature gradient may also dampen convection. Therefore, the main object of this paper is to show that the external constraint of rotation together with a suitable basic non-uniform temperature gradient suppresses Marangoni convection. The formulation of this problem in its simplest general form leads to a system of nonlinear equations with complicated boundary conditions which are difficult to deal with in general. Since nonlinear convection cannot be understood without reference to the growth of small disturbances, we consider in this paper only the linear theory with appropriate boundary conditions at the free surface, i.e. we predict the condition for the onset of convection in the presence of rotation and a non-uniform basic temperature gradient.

One of the earliest investigations on Marangoni convection was made by Pearson [4] under the assumptions of infinitesimally small amplitude analysis with capillary number tending to zero which means that the free surface does not deform. Pearson's analysis has been extended and refined by many authors (see refs. [1, 5, 6]). Recently, Rosenblat *et al.* [7, 8] studied nonlinear Marangoni convection in boundary layers and predicted sequences of transitions from one steady convective state to another as the Marangoni number is increased.

The above literature pertains to Marangoni convection subject to a uniform basic temperature gradient in the absence of an external constraint such as rotation. Vidal and Acrivos [9], Debler and Wolf [10] and Nield [11] discussed the effect of a non-uniform temperature gradient on the onset of Marangoni convection in the absence of rotation. Sarma [12] has examined the effect of rotation on instability in the presence of a uniform temperature gradient. He has illustrated the role of different boundary conditions on convection and the destabilizing character of the long wave disturbance at the fluid–fluid interface using a neutral stability curve based on analytical relations of the pertinent eigenvalue problem. Recently, Rudraiah [6] has discussed the combined effect of rotation and non-uniform basic temperature gradient on Marangoni convection, when one boundary is free and the other rigid with an adiabatic boundary condition, and has determined the condition for the onset of convection. The eigenvalues that he found were very low because he assumed that the third-order derivatives of w were zero at the boundaries, which is not valid. His trial function itself shows that these derivatives are not zero. Therefore, he is not able to recover the case of vanishing Taylor number.

In the present paper, we improve the results of Rudraiah [6] and extend the analysis to include the condition for a rigid and isothermal boundary which is useful for material processing in a microgravity environment. It will be shown that the basic temperature distribution may inhibit or augment convection depending on the nature of the temperature profile. The effect of rotation is, in general, to dampen Marangoni convection. Comparison of our analytical results with the numerical data of Vidal and Acrivos [9], in the absence of rotation, reveals that a single term Galerkin expansion used here gives reasonable results with minimum mathematics.

2. FORMULATION OF THE PROBLEM

Consider a viscous liquid which partially fills a thin horizontal layer extending to infinity in the two horizontal directions and rotating about a vertical axis with a constant angular velocity Ω. The mean depth of the liquid layer is d, its upper surface is open to an ambient gas. The liquid is assumed to behave like a Newtonian fluid with constant viscosity μ, and to conduct heat with constant thermal diffusivity κ and conductivity K. The liquid–gas interface has a surface tension σ which, following Pearson [4], can be assumed to vary linearly with temperature according to

$$\sigma = \sigma_0 - \sigma_1 \Delta T, \quad \Delta T = T_S - T_0. \qquad (1)$$

A Cartesian coordinate system (x, y, z) is used with the origin at the bottom of the boundary, ox parallel to the boundaries and oz coinciding with the axis of rotation so that ox, oy rotate about oz with the same angular velocity Ω. The lower boundary is assumed to be rigid and isothermal or adiabatic and the upper one (at $z = d$) free and adiabatic. Surface tension acts at the upper boundary where the usual stress balance applies (see ref. [13]). The liquid is assumed to be cooled from above by heat transfer to the gas. The basic equations in the bulk of the liquid are the energy-balance, Navier–Stokes and continuity equations with the Coriolis acceleration in the momentum equation. In their linearized and non-dimensionalized form [using d, d^2/κ, κ/d and $\Delta T/(aM^{1/2})$ as length, time, velocity and temperature scales, respectively], the equations for the perturbation field quantities are found to be

$$\frac{\partial T}{\partial t} - aM^{1/2} f(z) w = \nabla^2 T, \qquad (2)$$

$$\left(\frac{1}{Pr}\frac{\partial}{\partial t} - \nabla^2\right)\zeta - \mathscr{T}\frac{\partial w}{\partial z} = 0, \qquad (3)$$

$$\left(\frac{1}{Pr}\frac{\partial}{\partial t} - \nabla^2\right)^2 \nabla^2 w + \mathscr{T}^2 \frac{\partial^2 w}{\partial z^2} = 0, \qquad (4)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0. \qquad (5)$$

We note that the temperature scale has been chosen such that M appears symmetrically in the energy equation and the boundary condition rather than just in one equation. Though this choice of temperature scale is not essential for our purpose and has no physical implication, it enables a variational principle to be established for the present set of equations and the adjoint set. This leads to the conclusion that the eigenvalue M is stationary in the Galerkin method used in this paper.

In seeking the solutions of these equations we must satisfy certain boundary conditions for velocity and temperature. Those for velocity at the lower horizontal rigid boundary are the well-known no-slip conditions

$$w = 0, \quad \frac{\partial w}{\partial z} = 0, \quad \zeta = 0 \quad \text{at} \quad z = 0. \tag{6}$$

The boundary condition on temperature will depend on the nature of the boundary. If the lower plane is isothermal, then

$$T = 0 \quad \text{at} \quad z = 0, \tag{7}$$

which is a limiting approximation in the case where the boundary has a high heat conductivity and a large heat capacity. However, if the lower plane is adiabatic, then

$$\frac{\partial T}{\partial z} = 0 \quad \text{at} \quad z = 0. \tag{8}$$

The boundary conditions at the upper free surface are rather complicated in general and the shape of the free surface may change with rotation. The equation for the free surface S is

$$z = 1 + \eta(x, y, t). \tag{9}$$

Then the kinematic surface condition is

$$\frac{\partial \eta}{\partial t} + u\frac{\partial \eta}{\partial x} + v\frac{\partial \eta}{\partial y} + w = 0 \quad \text{on} \quad S. \tag{10}$$

To obtain the dynamic condition, we consider the balance of forces in a control volume V within the liquid in the form [13]

$$\int_V \mathbf{A}\, \mathrm{d}V + \int_S \tau \cdot \hat{n}\, \mathrm{d}S + \oint_{\partial l} \hat{n}_l \sigma\, \mathrm{d}l = 0, \tag{11}$$

where ∂l is a curve on S and $\hat{n}_l$ is the unit vector normal to ∂l. Applying a suitable limiting process, the dimensionless dynamic boundary conditions become [7]

$$MC(\tau \cdot \hat{n})_n - 2H(1 + MC(\eta - T)) = 0, \tag{12}$$

$$(\tau \cdot \hat{n})_t + \hat{n}(\hat{n} \times \nabla(\eta - T)) = 0. \tag{13}$$

The capillary number C is associated with the deflection of the free surface. We assume here that the deflection caused by rotation is small. Then $C \to 0$ corresponds to a flat surface [4, 7, 14] which makes the problem relatively simple and amenable to analytical treatment. In this limit equation (12) reduces to

$$H = \tfrac{1}{2}\nabla_1 \cdot \left\{ \frac{\nabla \eta}{(1 + (\nabla \eta)^2)^{1/2}} \right\} = 0. \tag{14}$$

A solution of equation (14), satisfying the conditions of contact angle and volume conservation is

$$\eta = 0, \tag{15}$$

representing an underformed free surface. In other words the surface deflection caused by rotation remains small when $C \ll 1$. In view of this, equation (12) is redundant and conditions (10) and (13) take the form

$$w = D^2 w + aM^{1/2} T = 0 \quad \text{at} \quad z = 1, \tag{16}$$

together with

$$\frac{\partial \zeta}{\partial z} = 0 \quad \text{at} \quad z = 1. \tag{17}$$

Since the free surface is assumed to be adiabatic, the condition on temperature is

$$\frac{\partial T}{\partial z} = 0 \quad \text{at} \quad z = 1. \tag{18}$$

3. CONDITION FOR THE ONSET OF MARANGONI CONVECTION

When the basic temperature gradient is uniform (i.e. $f(z) = 1$) exact solutions of equations (2)–(5) satisfying the boundary conditions (6)–(8) were obtained by Pearson [4] in the absence of rotation and by Sarma [12] in the presence of rotation. However, such exact solutions are difficult to find in the case of non-uniform temperature gradient, i.e. $f(z) \neq 1$. Therefore, in this paper we apply the Galerkin method as described in refs. [6, 11]. Rotation may give rise to oscillatory (i.e. overstable) motions only for small values of Pr and for particular values of $\mathscr{T}^2$ (see refs. [15, 16]). For other values of Pr and $\mathscr{T}^2$ the oscillatory motion, however, is not possible and the principle of exchange of stability is valid. In the absence of rotation Vidal and Acrivos [9] have shown that this principle is valid which means that the growth rate of the most dangerous disturbance changes from real negative to real positive values as M increases through its critical value [7]. Although the system (2) is not self-adjoint, we deal with those values of Pr and $\mathscr{T}^2$ which do not allow for oscillatory motions. Thus, all variables are independent of time.

Assuming the solution for w, T, ζ in the form $\phi(z) \exp(\mathrm{i}(lx + my))$, equations (2)–(5) after eliminating ζ take the form

$$(D^2 - a^2)^3 w + \mathscr{T}^2 D^2 w = 0, \tag{19}$$

$$(D^2 - a^2)T + aM^{1/2} f(z) w = 0. \tag{20}$$

To find the onset of Marangoni convection, we consider two situations. In the first both the boundaries are adiabatic and in the second the lower rigid plane is isothermal and the upper free and adiabatic.

3.1. *Both boundaries adiabatic*

The required boundary conditions, from equations (6) to (18), are

$$w = Dw = DT = 0 \quad \text{at} \quad z = 0, \tag{21}$$

$$w = D^2 w + aM^{1/2} T = DT = 0 \quad \text{at} \quad z = 1. \tag{22}$$

Multiplication of equation (19) by w, of equation (20) by T and integration of the resulting equations by parts with respect to z from 0 to 1, using the boundary conditions (21) and (22) and $w = Aw_1$, $T = BT_1$, where w_1 and T_1 are the trial functions, yields the following eigenvalue equation

$$M = -\frac{\langle (DT)^2 + a^2T^2\rangle}{a^2G}[\langle (D^3w)^2 + 3a^2(D^2w)^2 + (\mathscr{T}^2 + 3a^4)(Dw)^2 + a^6w^2\rangle + Dw(1)D^4w(1) + D^2w(0)D^3w(0)], \quad (23)$$

$$G = [D^3w(1)T(1) + 3a^2Dw(1)T(1)]\langle f(z)wT\rangle.$$

We select the trial functions

$$w_1 = z^2(1-z), \quad T_1 = 1, \quad (24)$$

such that they satisfy all the boundary conditions except the one given by $D^2w + aM^{1/2}T = 0$ at $z = 1$ but the residual from this is included in a residual from the differential equation. Substituting equation (24) into equation (23) and performing the integration, we get

$$M = \frac{\alpha^3 + 24\alpha^2 + 1260\alpha + 14\mathscr{T}^2 + 2520}{315(\alpha+2)\langle f(z)(z^2 - z^3)\rangle}, \quad (25)$$

where $\alpha = a^2$.

For any given $f(z)$, M attains its minimum value at $\alpha_c = a_c^2$, α_c being the root of the cubic equation

$$\alpha^3 + 24\alpha^2 + 84\alpha - 7\mathscr{T}^2 = 0. \quad (26)$$

If $\mathscr{T}^2 = 0$, then $\alpha_c = 0$, which is exactly the one given by Nield [11] in the absence of rotation. The variation of α_c with $\mathscr{T}^2$ is computed for different values of $\mathscr{T}^2$ and the results are shown in Table 1. From this it is clear that as in the Rayleigh–Bénard convection [15] the wavelength decreases with increasing $\mathscr{T}^2$ and hence the effect of rotation even in the case of Marangoni convection is to contract the cells. When the layer of liquid is heated from below at a constant rate, the non-uniform temperature gradient $f(z)$ is not only non-negative but also decreases monotonically. Thus, we are interested in knowing which temperature profile gives the least M subject to $f(z) \geq 0$.

3.1.1. *Case 1 : linear profile.* The basic temperature gradient is uniform, i.e. $f(z) = 1$, and equation (25) takes the form

$$M_1 = \frac{4(\alpha^3 + 42\alpha^2 + 1260\alpha + 14\mathscr{T}^2 + 2520)}{105(\alpha+2)}. \quad (27)$$

We note that, when $\mathscr{T}^2 = 0$, $\alpha_c = 0$ and hence equation (27) gives $(M_c)_1 = 48$ which is the known exact value at the critical Marangoni number for this case. In the presence of rotation, $(M_c)_1$ is computed for different values of $\mathscr{T}^2$ and the results are shown in Table 1.

3.1.2. *Case 2 : piecewise linear profile for heating from below.* For a piecewise linear profile [11]

$$f(z) = \begin{cases} 1/\varepsilon & \text{for} \quad 0 \leq z < \varepsilon, \\ 0 & \text{for} \quad \varepsilon < z \leq 1. \end{cases} \quad (28)$$

Then equation (25) using equation (28) becomes

$$M_2 = \frac{M_1}{4\varepsilon^2 - 3\varepsilon^3}, \quad (29)$$

and the corresponding critical value is

$$(M_c)_2 = \frac{(M_c)_1}{\text{Max}\,(4\varepsilon^2 - 3\varepsilon^3)}. \quad (30)$$

Then as ε increases from 0 to 1, $(M_c)_2$ decreases from infinity to a minimum value of

$$(M_c)_2 = \frac{(M_c)_1}{1.0534977}, \quad (31)$$

attained at $\varepsilon = 0.8889$, and then increases to $(M_c)_1$ at $\varepsilon = 1$. $(M_c)_2$ given by equation (31) is computed for different values of $\mathscr{T}^2$ and the results are shown in Table 1.

3.1.3. *Case 3 : piecewise linear profile for cooling from above.* When the liquid layer is cooled from above at a constant rate, the temperature gradient is not only non-negative but also monotonically decreasing. In the absence of Coriolis force Vidal and Acrivos [9] have discussed convection numerically using this temperature profile. Following them we consider a piecewise linear profile

$$f(z) = \begin{cases} 0 & 0 \leq z < 1-\varepsilon, \\ \varepsilon^{-1} & 1-\varepsilon < z \leq 1. \end{cases} \quad (32)$$

Substituting this into equation (25), we get

$$M_3 = \frac{M_1}{3\varepsilon^3 - 8\varepsilon^2 + 6\varepsilon}, \quad (33)$$

and the corresponding critical Marangoni number is

$$(M_c)_3 = \frac{(M_c)_1}{\text{Max}\,(3\varepsilon^3 - 8\varepsilon^2 + 6\varepsilon)}. \quad (34)$$

As ε increases from 0 to 1, $(M_c)_3$ decreases from $+\infty$ to a minimum value of

$$(M_c)_3 = \frac{(M_c)_1}{1.380} \quad \text{at} \quad \varepsilon = 0.5375, \quad (35)$$

Table 1. Values of critical Marangoni and wave numbers for various values of $\mathscr{T}^2$ in the case of adiabatic boundaries

$\mathscr{T}^2$	a_c	$(M_c)_1$	$(M_c)_2$	$(M_c)_3$	$(M_c)_4$	$(M_c)_5$	$(M_c)_6$
0	0.0000	48.0000	45.5625	34.7826	40.0000	60.0000	26.9996
10^{-1}	0.0912	48.0266	45.5877	34.8018	40.0221	60.0332	27.0146
10^0	0.2854	48.2613	45.8105	34.9719	40.2177	60.3266	27.1466
10^1	0.8318	50.2704	47.7176	36.4278	41.8920	62.8380	28.2767
10^2	1.9321	61.5377	58.4127	44.5925	51.2814	76.9221	34.6145
10^3	3.5653	107.1449	101.7039	77.6412	89.2874	133.9311	60.2682

and then increases to $(M_c)_1$ at $\varepsilon = 1$. The eigenvalue $(M_c)_3$ given in equation (35) is computed for different values of $\mathscr{T}^2$ in Table 1. Comparing equations (31) and (35) we find that cooling from above is, as expected, more effective in reducing the eigenvalues than heating from below.

3.1.4. *Case 4: parabolic basic temperature profile.* In the absence of rotation Debler and Wolf [10] have considered such a profile in which the basic temperature gradient is zero at the lower boundary and $f(z) = 2z$. In the case of porous media Rudraiah *et al.* [17, 18] have also used this temperature distribution. Even in the presence of Coriolis force, the parabolic basic temperature distribution leads to $f(z) = 2z$. In this case equation (25) takes the form

$$M_4 = \frac{M_1}{1.2}, \tag{36}$$

and the corresponding critical value is

$$(M_c)_4 = \frac{(M_c)_1}{1.2}. \tag{37}$$

Values of $(M_c)_4$ computed for different values of $\mathscr{T}^2$ are shown in Table 1.

3.1.5. *Case 5: inverted parabolic temperature profile.* For an inverted parabolic profile $f(z) = 2(1-z)$ (see refs. [19, 20] and the corresponding critical Marangoni number is

$$(M_c)_5 = 1.25(M_c)_1. \tag{38}$$

Comparing this with the earlier results, we find, as expected on physical grounds, that the inverted parabolic basic temperature profile is more stabilizing. Thus, it can be used to suppress the onset of Marangoni convection. This type of temperature profile is most suitable to produce large crystals of uniform properties and also to manufacture materials with unique properties, under a microgravity environment.

3.1.6. *Case 6: delta function temperature profile.* We consider the step function profile in which the basic temperature drops suddenly by an amount ΔT at $z = \varepsilon$, but is otherwise uniform. It is of the form

$$f(z) = \delta(z-\varepsilon), \tag{39}$$

where ε is the value of z at which wT is maximum and δ denotes the Dirac delta function. In this case equation (25) becomes

$$M_6 = \frac{M_1}{12(\varepsilon^2-\varepsilon^3)}, \tag{40}$$

and the values of the critical Marangoni number

$$(M_c)_6 = \frac{(M_c)_1}{12\ \mathrm{Max}(\varepsilon^2-\varepsilon^3)}, \tag{41}$$

computed from this are listed in Table 1.

We find that the most unstable temperature profile, for which $f(z) \geqslant 0$ everywhere, is the Dirac delta function profile for which the step occurs at the level at which w is maximum, since T is constant in our approximation.

3.2. *Lower boundary rigid and isothermal and upper boundary free and adiabatic.*

In Section 3.1 we have discussed the situation when both boundaries are adiabatic. The imposition of an adiabatic condition at the rigid surface is very restrictive and in many practical problems particularly in laboratory problems one usually chooses perfectly conducting, i.e. isothermal rigid surfaces in order to supply heat by heating from below. We consider such a situation in this section. The boundary conditions are

$$w = Dw = T = 0 \quad \text{at} \quad z = 0, \tag{42}$$

together with condition (22) at the upper surface. These are satisfied by the trial functions

$$w_1 = z^2(1-z^2) \quad \text{and} \quad T_1 = z(1-\tfrac{1}{2}z). \tag{43}$$

Substitution of equation (43) into equation (23) and integration leads to

$$M = 4\{[4\alpha^4+208\alpha^3+8433\alpha^2+(57\,645+66\mathscr{T}^2)\alpha + 94\,500+165\mathscr{T}^2]/[14\,175\alpha(\alpha+4)\langle fwT\rangle]\}. \tag{45}$$

For any given $f(z)$, M attains its minimum value, when $\alpha = \alpha_c = a_c^2$ given by the root of the equation

$$\alpha^5+32\alpha^4+208\alpha^3-(8.25\mathscr{T}^2+2989.125)\alpha^2 -(41.25\mathscr{T}^2+23\,625)\alpha-82.5\mathscr{T}^2-47\,250 = 0. \tag{46}$$

Using the procedure explained in Section 3.1 $(M_c)_i$ values are computed for different temperature profiles $f(z)$ and the result is shown in Table 2.

Table 2. Values of critical Marangoni and wave numbers for various values of $\mathscr{T}^2$ in the case of one boundary adiabatic and the other isothermal

$\mathscr{T}^2$	a_c	$(M_c)_1$	$(M_c)_2$	$(M_c)_3$	$(M_c)_4$	$(M_c)_5$	$(M_c)_6$
0	3.113	64.85	62.798	38.618	48.903	96.229	30.747
10^{-1}	3.113	64.85	62.798	38.618	48.903	96.229	30.747
10^0	3.114	64.88	62.827	38.636	48.926	96.274	30.762
10^1	3.128	65.16	63.098	38.803	49.137	96.689	30.894
10^2	3.249	67.85	65.703	40.405	51.166	100.681	32.170
10^3	4.010	88.74	83.067	52.845	66.919	131.679	42.074
10^4	6.032	196.36	190.146	116.932	148.075	291.373	93.100
10^5	9.248	667.49	646.366	397.490	503.353	990.469	316.477

Table 3. Critical values of $(M_c)_3$ for various values of the thermal depths

	$\mathscr{T}^2 = 0$			$\mathscr{T}^2 = 10$	$\mathscr{T}^2 = 10^2$	$\mathscr{T}^2 = 10^3$
ε	Present analysis $(M_c)_3$ for $a_c = 0$	Vidal and Acrivos [9] M_c	a_c	$(M_c)_3$ for $a_c = 0.8318$	$(M_c)_3$ for $a_c = 1.9321$	$(M_c)_3$ for $a_c = 3.5653$
0.0						
0.05	171.1993	104.29	3.3	179.2970	219.4835	382.1485
0.1	91.7782	64.56	2.7	96.1193	117.6629	204.8659
0.4	36.5854	36.00	1.0	38.3158	46.9037	81.6653
0.5	34.9091	34.9	0.1	36.5603	44.7547	77.9236
0.8	39.4737	39.5	0.001	41.3408	50.6067	88.1126
1.0	48.0000	48.0	0.0	50.2704	61.5377	107.1449

4. DISCUSSION AND CONCLUSIONS

The single term Galerkin expansion provides a quick means for obtaining the condition for the onset of convection due to surface tension, when the upper free surface is flat and adiabatic while the lower rigid surface may be adiabatic or isothermal. The results (27), (31), (35), (37), (38), (41) and (45) give the critical Marangoni number for the corresponding critical wave number. Both vary with Taylor number. Numerical values are listed in Tables 1 and 2. From these it is clear that the wave number increases with increasing $\mathscr{T}^2$ having the asymptotic behaviour

$$a_c^2 \to 1.913\mathscr{T}^{2/3} \quad \text{as} \quad \mathscr{T}^2 \to \infty, \qquad (47)$$

when both the boundaries are adiabatic and

$$a_c^2 \to 2.417\mathscr{T}^{2/5} \quad \text{as} \quad \mathscr{T}^2 \to \infty, \qquad (48)$$

when the upper boundary is adiabatic and the lower boundary is isothermal. We conclude that the critical non-dimensional wave number depends crucially on $\mathscr{T}^2$ and on thermal conditions but is independent of the nature of the basic temperature profile. Tables 1 and 2 reveal that the critical Marangoni number increases with increasing $\mathscr{T}^2$ and the asymptotic behaviour depends on the nature of the basic temperature profile, $\mathscr{T}^2$ and on the thermal conditions. For example, when the boundaries are adiabatic and the basic temperature profile is linear, i.e. $f(z) = 1$ the asymptotic behaviour of M_c is

$$M_c \to 0.2788\mathscr{T}^{4/3} \quad \text{as} \quad \mathscr{T}^2 \to \infty, \qquad (49)$$

which is analogous to the one given by Veronis [15] for Bénard convection. In the case of piecewise linear profile [cf. equation (28)] the asymptotic behaviour for M_c is

$$M_c \to 0.2646\mathscr{T}^{4/3}, \qquad (50)$$

attained at $\varepsilon = 0.8889$. Further, from Tables 1 and 2 it follows that

$$(M_c)_6 < (M_c)_3 < (M_c)_4 < (M_c)_2 < (M_c)_1 < (M_c)_5. \qquad (51)$$

Thus in both thermal conditions, the most unstable basic temperature profile is the one for which the temperature gradient is a Dirac delta function and the most stable one is the inverted parabola.

When the basic temperature profile is linear, the condition for the onset of Marangoni convection in the presence of Coriolis force was investigated by Sarma [12]. The results of Table 1 for $(M_c)_1$ are compared with those of Sarma (his Fig. 4) and a good agreement is found. Table 2 shows that when $f(z) = 1$ and $\mathscr{T}^2 \to 0$, the value 64.85 is close to the value 67 given by Finlayson [5]. The difference in M_c is due to the difference in the critical wave number, which can be improved by considering higher order terms. But our aim was to show that even this approximate method of using a single term Galerkin expansion gives results close to the exact values and is useful to understand the physics of the problem. A further test of the validity of our results is made in comparing the approximate values of M_3 given by (3.19) with the numerical data of Vidal and Acrivos [9] in the absence of Coriolis force, see Table 3.

Table 3 shows that equation (23) gives the upper bound on M_c which agrees well with the numerical results of Vidal and Acrivos [9] when $\mathscr{T}^2 = 0$ and $\varepsilon \geqslant 0.4$. The deviation for $\varepsilon < 0.4$ may be due to the fact that the critical wave number in the case of Vidal and Acrivos [9] differs markedly from zero while it has negligible difference for $\varepsilon \geqslant 0.4$.

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CONVECTION MARANGONI DANS UNE COUCHE FLUIDE EN ROTATION AVEC UN GRADIENT NON UNIFORME DE TEMPERATURE

Résumé— L'apparition de la convection Marangoni due aux gradients de tension interfaciale dans une mince couche horizontale de fluide est étudiée au moyen d'une analyse linéaire de stabilité qui suppose qu'une frontière est libre et adiabatique tandis que l'autre est rigide, adiabatique ou isotherme. Une technique de Galerkin est utilisée pour obtenir les valeurs propres qui sont calculées par ordinateur. La force de Coriolis et le profil de température parabolique inversé sont appropriés pour supprimer la convection Marangoni dans un environnement de microgravité. Les résultats analytiques s'accordent avec les résultats numériques de Vidal et Acrivos [*Physics Fluids* **9**, 615–616 (1966)] en l'absence de la force de Coriolis.

MARANGONI-KONVEKTION IN EINER ROTIERENDEN FLUIDSCHICHT MIT VERÄNDERLICHEM TEMPERATURGRADIENTEN

Zusammenfassung—Es wird das Einsetzen der Marangoni-Konvektion in einer dünnen horizontalen Fluidschicht mittels linearer Stabilitätsanalyse unter der Annahme untersucht, daß eine der begrenzenden Oberflächen keine Schubspannungen überträgt und adiabatisch ist, während die andere der Haftbedingung genügt und entweder adiabatisch oder isotherm ist. Ein Galerkin-Verfahren führt zu Eigenwerten, die numerisch berechnet werden. Der Coriolis-Effekt und ein invertiertes parabolisches Grundprofil der Temperatur sind geeignete Mittel, um z.B. das Kristallwachstum bei verschwindender Schwerkraft zu beeinflussen, da sie die Marangoni-Konvektion unterdrücken. Die Ergebnisse sind in guter Übereinstimmung mit denen von Vidal und Acrivos [*Physics Fluids* **9**, 615–616 (1966)] in Abwesenheit von Coriolis-Kräften.

КОНВЕКЦИЯ МАРАНГОНИ ВО ВРАЩАЮЩЕМСЯ СЛОЕ ЖИДКОСТИ С НЕОДНОРОДНЫМ ГРАДИЕНТОМ ТЕМПЕРАТУРЫ

Аннотация—Методом линейной устойчивости рассматривается установление конвекции Марангони, обусловленной градиентами поверхностного натяжения в тонком горизонтальном слое жидкости в предположении, что одна из поверхностей свободная и адиабатическая, а другая – жесткая адиабатическая или изотермическая. Методом Галеркина численно получены собственные значения. Аналитические результаты хорошо согласуются с численными результатами Видала и Акривоса [9] в отсутствие силы Кориолиса.